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TWO HYDRAULIC METHODS TO EXTRACT THE *n*th ROOT OF ANY NUMBER.

By DR. ARNOLD EMCH, University of Colorado.

1. The first method is based upon the construction of a vessel which is bounded by such a surface of revolution that the weight of the water displaced by the surface is always equal to the *n*th power of the depth of immersion. Suppose that $x=f(y)$ is the equation of a meridian of the surface, Fig. I, for which OY is the axis of revolution, and that the meridian touches the line OX at O. Using one foot as the unit of measure and designating by w the weight of a cubic foot of water which may be assumed as 62.5 pounds, the weight of the displaced water, when the depth of immersion is y, is

$$W=\pi.w\int_0^y x^2.dy=\pi.w\int_0^y f^2(y)dy \;\ldots.(1),$$

and this shall equal y^n. Hence

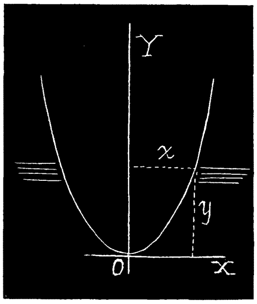

Fig. 1.

$$\pi w\int_0^y f^2(y)dy=y^n\ldots.(2),$$

from which $x=f(y)$ is easily found:

$$x=f(y)=\sqrt{\frac{n}{\pi.w}}y^{\frac{1}{2}(n-1)}\;\ldots.(3),\quad\text{or}\quad y=\sqrt{\frac{\pi.w}{n}}x^{2/(n-1)}\ldots.(4).$$

Giving n successively the values 1, 2, 3, ... we find for the equations of the meridians

$$x=\sqrt{\frac{1}{\pi.w}}=0.05052\ldots.(5),$$

$$y=\sqrt{\frac{\pi.w}{2}}x^2=9.9083.x^2\ldots.(6),$$

$$y=\sqrt{\frac{\pi.w}{3}}x=8.0901.x\ldots.(7),$$

etc., (unit=1 foot)

so that for the first, square, and cube roots, the surfaces are respectively a cylinder, a paraboloid and a cone of rotation. Now by the principle of Archimedes the weight of the water displaced is equal to the weight of the vessel. Hence, if this weight is Q (in pounds) and if we want to extract the nth root of a given number N, we add to Q a weight P in such a manner that $Q+P=N$. After the vessel is in equilibrium, there is

$$Q+P=N=W=y^n$$

and consequently,

$$y=\sqrt[n]{N}\ldots.(8).$$

The depth of immersion, y, is therefore the nth root of the number N.

In Fig. 2 an apparatus for the extraction of the square root has been sketched. The shape of the paraboloid of rotation is determined by equation (6) and corresponds to the wetted surface of the vessel. Suppose P is the weight to be added at A to make $Q+P=N$, then the difference of the water-levels before and after immersion will be equal to y or the square root of N. The difference in levels may be measured by the hook-guage G. In the mechanical execution of this apparatus it is not difficult to secure any reasonable precision and to eliminate errors of construction and observation.

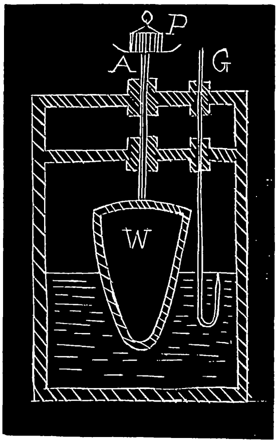

Fig. 2.

2. A second method to extract the nth root of any number may be based upon the problem to find the time necessary of emptying a vessel of given form through a small orifice at the bottom. We assume again a surface of revolution and refer it to the same axes as in the previous problem. Then the time dt to lower the level of the water in the vessel by dy units is expressed by the formula,

$$dt=\frac{\pi.x^2.dy}{a\sqrt{(2gy)}}=\frac{\pi f^2(y)dy}{a\sqrt{(2gy)}}*\ldots.(9),$$

where a is the cross-section of the orifice.

Hence the time of emptying the vessel from the original level y is

$$t=\frac{\pi}{a\sqrt{(2g)}}\int_y^0\frac{f^2(y)dy}{\sqrt{y}}\ldots\ (10).$$

We shall now determine $f(y)$ in such a manner that

$$t=\sqrt[n]{y}=\frac{\pi}{a\sqrt{(2g)}}\int_y^0\frac{f^2(y)dy}{\sqrt{y}}\ldots.(11).$$

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*See Merriman's Hydraulics, page 56.

From (11) we find for $x=f(y)$ the expression

$$x=\sqrt{\frac{a\sqrt{(2g)}}{n.\pi}}.y^{(2-n)/4n}\ldots.(12),$$

which for $n=1, 2, 3, \ldots.$ *i. e.*, for the first, square, and cube-root, becomes

$$x=\sqrt{\frac{a\sqrt{(2g)}}{\pi}}.y^{\frac{1}{4}}\ \ldots.(13),$$

$$x=\sqrt{\frac{a\sqrt{(2g)}}{2\pi}}\ldots.(14),$$

$$x=\sqrt{\frac{a\sqrt{(2g)}}{2\pi}}.y^{-\frac{1}{12}}\ldots.(15),$$

respectively. The shape of the vessel in case of square-roots is therefore cylindrical, and if the radius of the cylinder is made according to (14) *the time of emptying the cylindrical vessel will be equal to the square root of the original depth of water.*

The physical conditions of the problem make it clear that the first method is more accurate. The determination of the time in the second method is liable to be affected with an error of a higher order than those occuring in the statical extraction of a root.

In the near future the author shall publish a remarkably simple extension of the first method to the solution of an equation of the form

$$a_0+a_1x+a_2x^2+a_3x^3+\ldots.+a_nx^n=0.$$

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

133. Proposed by **COOPER D. SCHMITT, A. M.**, Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

In Wentworth's Arithmetic he gives a formula $\frac{21}{40}(d^2-2d)$ for calculating the number of board feet in a log 10 feet long, when d is the diameter in inches. How is this rule derived?